Pipe Flow

Fully Developed Laminar Flow:

Dimensionless analysis

$$\Delta p = f\left(\bar{v}, \ell, D, \mu\right) \quad \rightarrow \quad \frac{D\Delta p}{\mu \bar{v}} = \phi\left(\frac{\ell}{D}\right)$$ Eq (8.17)

Assumption: $\Delta p \sim \ell$

$$\frac{D\Delta p}{\mu \bar{v}} = C'\left(\frac{\ell}{D}\right)$$

$$Q = A\bar{v} = \frac{\pi D^4}{4C'\mu\ell}\Delta p$$ Eq (8.18)

The constant $C' = 32$ for a round pipe

Pipe Flow

Fully Developed Laminar Flow:

Dimensionless analysis

$$\Delta p = C'\left(\frac{\ell}{D}\right)\frac{\mu\bar{v}}{D}$$

Making all dimensionless:

$$\frac{\Delta p}{\frac{1}{2}\rho\bar{v}^2} = \frac{2C'}{\mathrm{Re}}\left(\frac{\ell}{D}\right) = f_D\left(\frac{\ell}{D}\right)$$

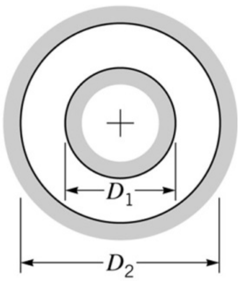
f_D is the Darcy friction factor

Pipe Flow

Fully Developed Laminar Flow:

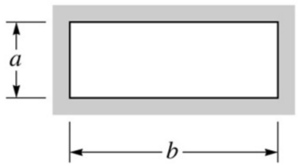
I. Concentric Annulus

$D_h = D_2 - D_1$



II. Rectangle

$D_h = \frac{2ab}{a+b}$



$$f_D = \frac{2C'}{\text{Re}} = \frac{C}{\text{Re}}$$

$C = 64$ for a round pipe

For noncircular ducts:

$$f_D = \frac{C}{\text{Re}_h}$$

where $$\text{Re}_h = \frac{\rho \bar{v} D_h}{\mu}$$

$$D_h = \frac{4A}{P}$$

A = area; P = perimeter

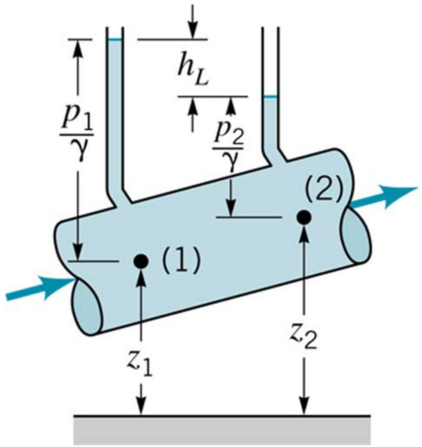
D_h = hydraulic diameter

Pipe Flow

Fully Developed Laminar Flow:

Energy consideration

$$\frac{\Delta p}{\gamma}+\frac{\Delta\alpha\bar{v}^2}{2g}+\Delta z=h_L$$



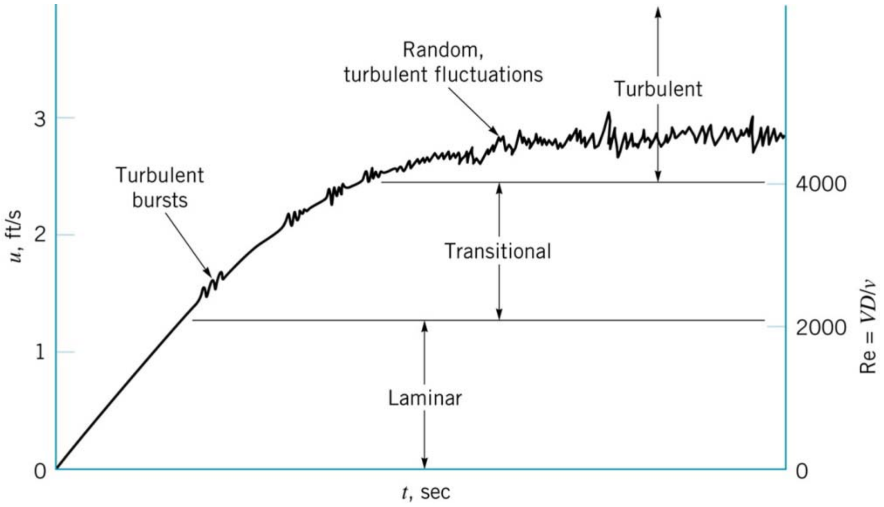
Fully developed flow: $\frac{\Delta\alpha\bar{v}^2}{2g}=0$

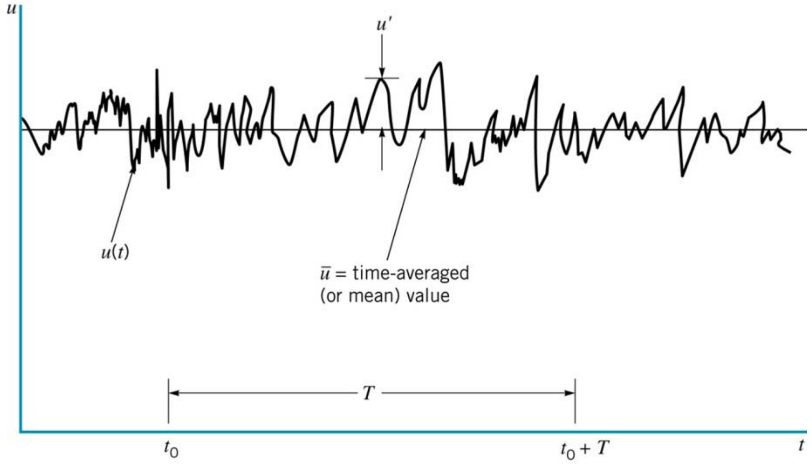
$$\left.\begin{array}{l}\dfrac{\Delta p+\gamma\Delta z}{\ell}=\dfrac{4\tau_w}{D}\\[2ex]\dfrac{\Delta p}{\gamma}+\Delta z=h_L\end{array}\right\}\rightarrow h_L=\frac{4\tau_w\ell}{\gamma D}$$

Eq (8.23)

Pipe Flow

Transition from Laminar to Turbulent Flow:





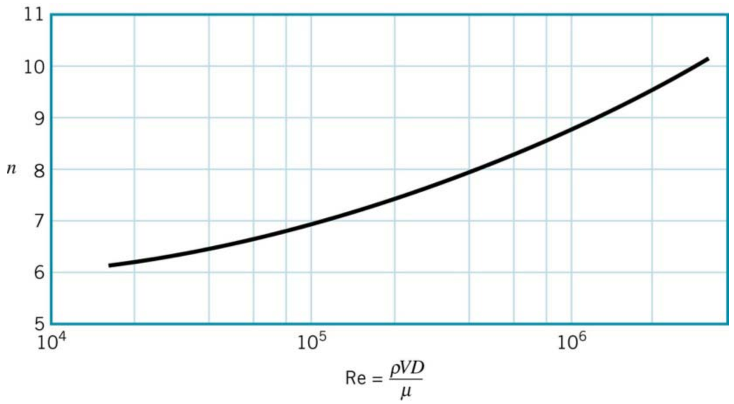
Pipe Flow

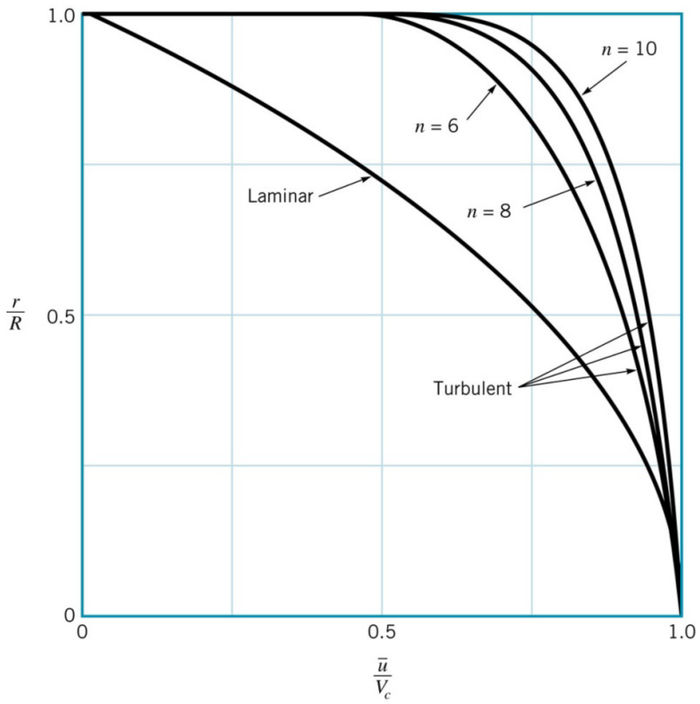
Fully Developed Turbulent Flow:

Power-law velocity profile:

$$\bar{u}(r) = v_C\left[1-\left(\frac{r}{R}\right)\right]^{\frac{1}{n}}$$

Eq (8.31)





Approximation for practical flow: $n = 7$