

Pipe Flow

Laminar or Turbulent flow:

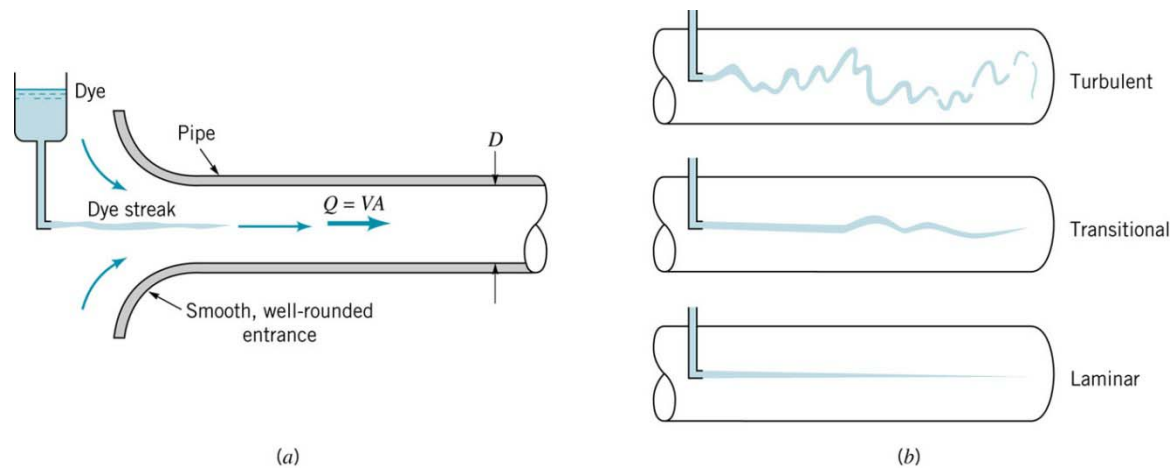
$$\text{Re} = \frac{\rho v D}{\mu}$$

Laminar: $\text{Re} < 2100$

Transitional: $2100 < \text{Re} < 4000$

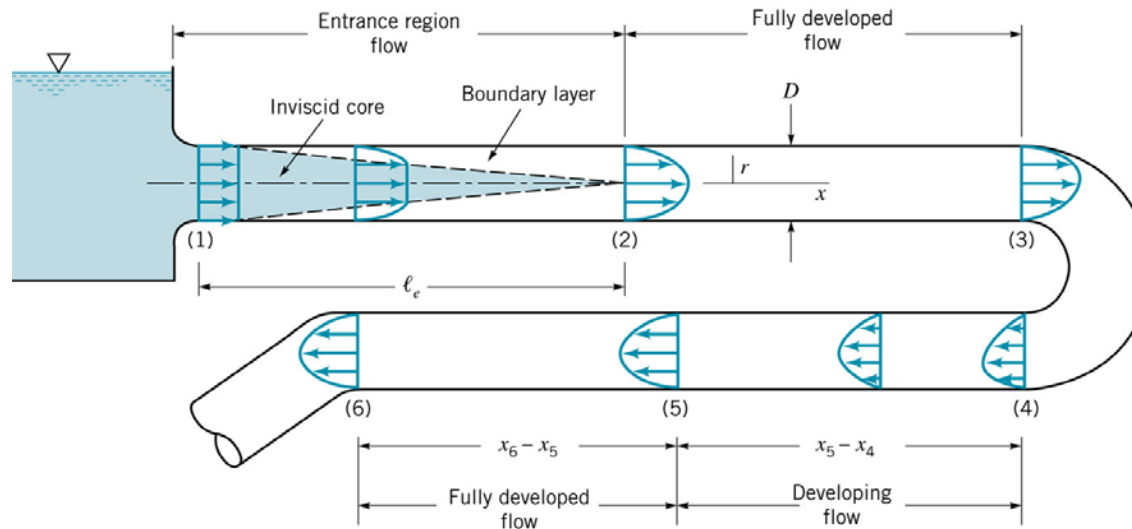
Turbulent: $\text{Re} > 4000$

Practical engineering problems $10^4 < \text{Re} < 10^5$



Pipe Flow

Entrance Region and Fully Developed Flow:



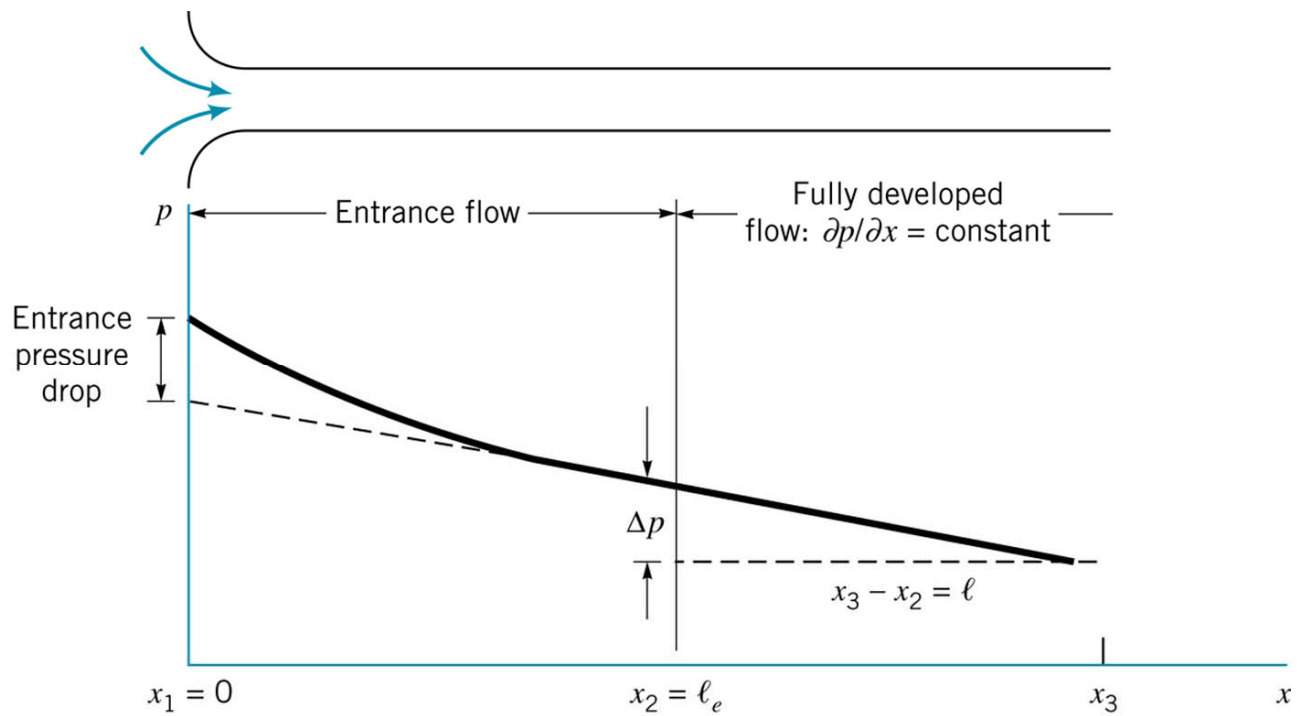
Laminar flow: $\frac{\ell_e}{D} = 0.06 \text{Re}$ Eq (8.1)

Turbulent flow: $\frac{\ell_e}{D} = 4.4 \text{Re}^{\frac{1}{6}}$ Eq (8.2)

For practical engineering problems:

$$20D < \ell_e < 30D$$

Pipe Flow

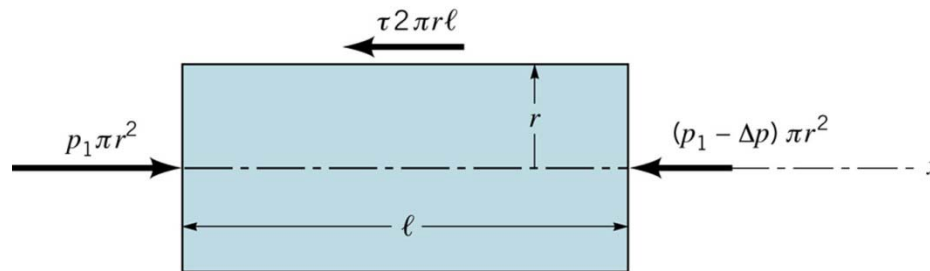


Pipe Flow

Newton's law: $\mathbf{F} = m\mathbf{a}$

Fully developed horizontal pipe flow is merely a balance between pressure and viscous forces:

$$\Delta p \cdot \pi r^2 = \tau(2\pi r \ell)$$

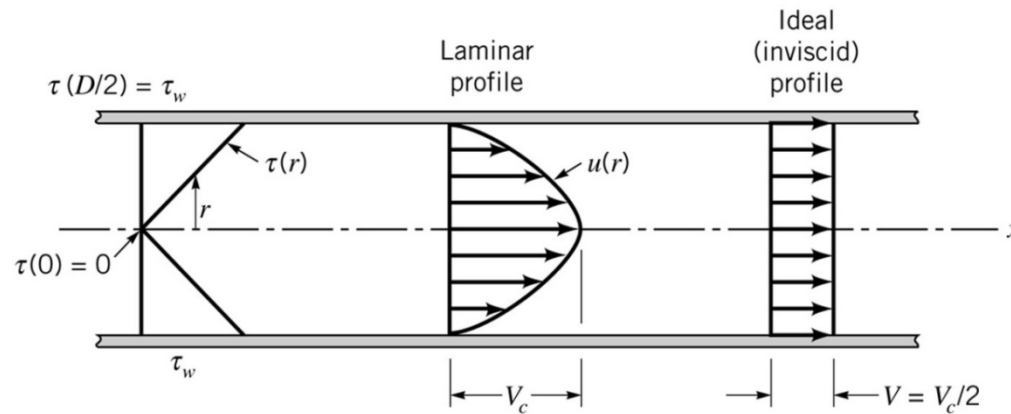


$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} = \text{constant} \quad \text{Eq (8.3)}$$

$$\Delta p = \frac{4\ell \tau_w}{D} \quad \text{Eq (8.5)}$$

Pipe Flow

Fully Developed Laminar Flow:



Laminar flow:

$$\left. \begin{aligned} \frac{\Delta p}{\ell} &= \frac{2\tau}{r} \\ \tau &= -\mu \frac{du}{dr} \end{aligned} \right\} \rightarrow \frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell} \right) r$$

Pipe Flow

Fully Developed Laminar Flow:

Velocity profile:

$$u(r) = \frac{D^2 \Delta p}{16\mu l} \left[1 - \left(\frac{2r}{D} \right)^2 \right] = v_C \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad \text{Eq (8.7)}$$

Flow rate:

$$\begin{aligned} Q &= \int u(r) dA = \bar{v} A = \frac{1}{2} v_C (\pi R^2) \\ &= \frac{\pi D^4}{128\mu l} \Delta p \quad \text{Eq (8.9)} \end{aligned}$$

Poiseuille's law

Nonhorizontal pipe:

$$Q = \frac{\pi D^4}{128\mu l} (\Delta p - \gamma l \sin \theta)$$

Eq (8.12)