Confined flows

Conservation of mass:

$$\dot{m} = \frac{dm}{dt} = \frac{d(\rho V)}{dt} = \text{ constant}$$

$$\dot{m}_{in} = \dot{m}_{out}$$

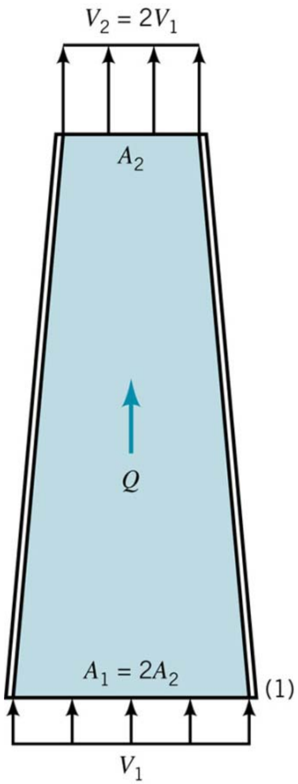
If the density does not change with time:

$$\dot{m} = \rho \frac{dV}{dt} = \rho Q = \text{ constant}$$

The volume flow rate: $Q = \frac{dV}{dt} = Av$

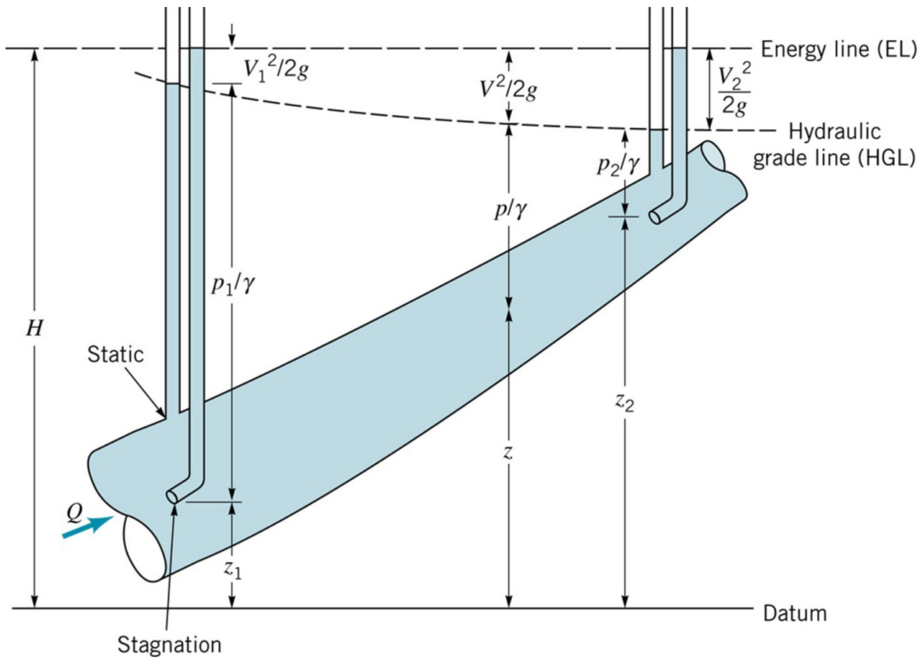
If the density is constant throughout space:

$$A_{in} v_{in} = A_{out} v_{out} \qquad (\textit{continuity equation})$$



Eq (3.19)

Energy line



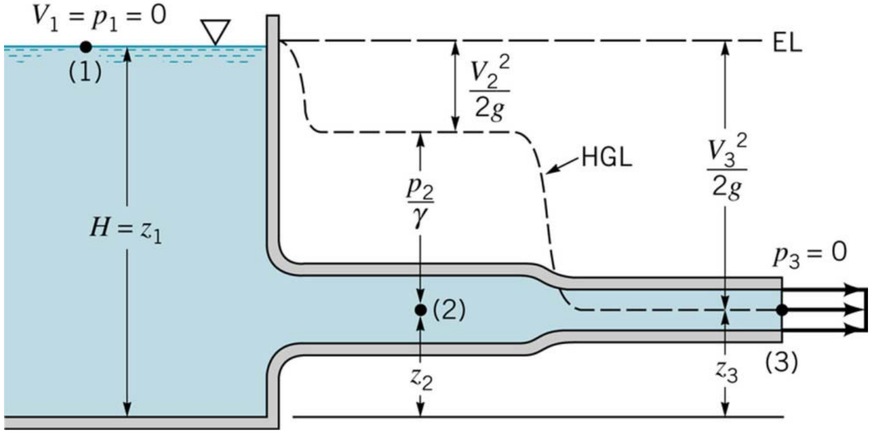
Total head available to the fluid:

$$H = \frac{p}{\gamma} + z + \frac{1}{2}\frac{v^2}{g}$$

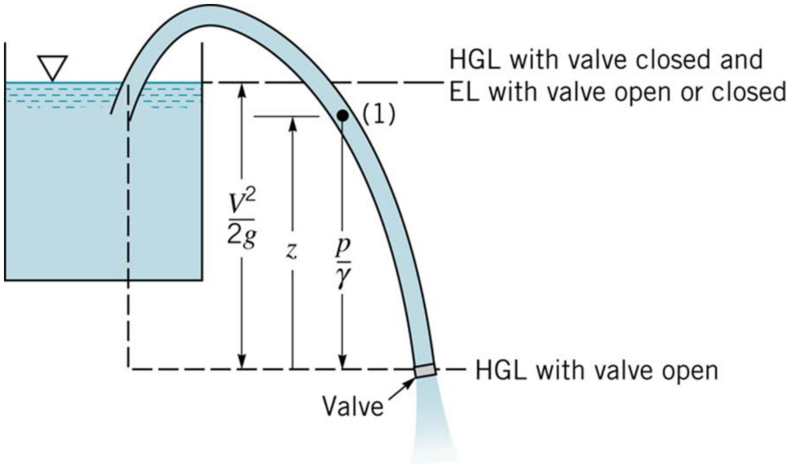
represented by the energy line (EL)

Hydraulic grade line (HGL) represents pressure and elevation heads only (= *piezometric* head).

Energy line

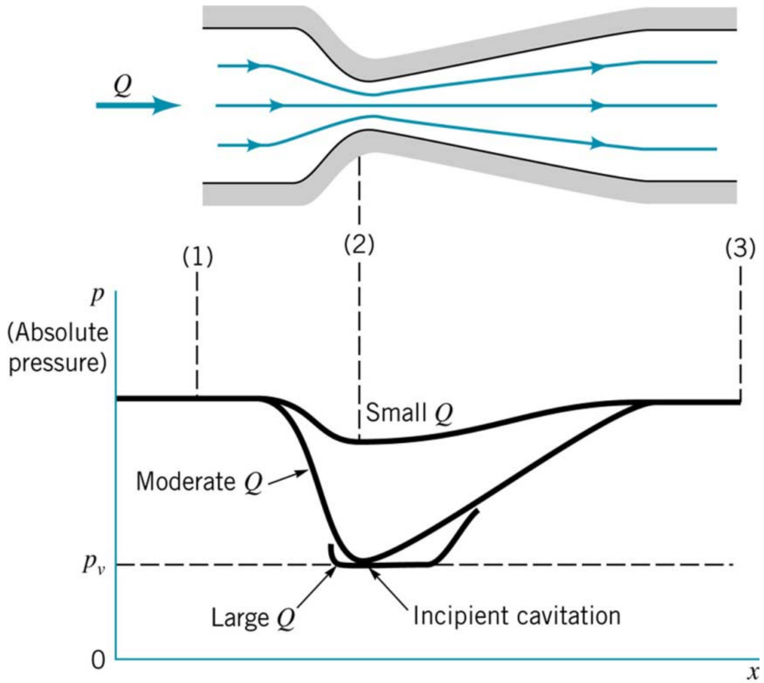


$V_1 = p_1 = 0$
(1)
$H = z_1$
EL
$\frac{V_2^2}{2g}$
HGL
$\frac{V_3^2}{2g}$
$\frac{p_2}{\gamma}$
$p_3 = 0$
(2)
z_2
z_3
(3)



HGL with valve closed and
EL with valve open or closed
(1)
$\frac{V^2}{2g}$
z
$\frac{p}{\gamma}$
HGL with valve open
Valve

Cavitation



$$A_1 v_1 = A_2 v_2$$

$$A_1 > A_2 \rightarrow v_1 < v_2$$

Along the center streamline: $z_1 = z_2$

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_1 < v_2 \rightarrow p_2 < p_1$$

If $p_2 = p_v$, *cavitation* occurs at in the neck

Restrictions on Bernoulli equation

$$\frac{p}{\rho} + gz + \tfrac{1}{2}v^2 = \text{ constant along streamline}$$

for *steady* flow of *<u>incompressible</u>* and *inviscid* fluid

If the fluid is compressible

$$\int \frac{dp}{\rho} + gz + \tfrac{1}{2}v^2 = \text{ constant along a streamline}$$

for *steady* flow of *<u>compressible</u>* and *inviscid* fluid

However, if the *pressure change* is small, then the density will not change much. In this case approximation with incompressible fluid is good enough.

Restrictions on Bernoulli equation

$$\Delta p + \gamma \Delta z + \tfrac{1}{2}\rho \Delta v^2 = 0$$

for <u>*steady*</u> flow of *incompressible* and *inviscid* fluid

If the flow is unsteady

The velocity of the flow is also a function of time along a streamline: $v(s,t)$

$$\left(\frac{dp}{ds}\right) + \left(\gamma \frac{dz}{ds}\right) = -\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s}\right)$$

$$\Delta p + \gamma \Delta z + \tfrac{1}{2}\rho \Delta v^2 = \rho \int_{s_1}^{s_2} \left(\frac{\partial v}{\partial t}\right) ds$$

for <u>*unsteady*</u> flow of *incompressible* and *inviscid* fluid

Restrictions on Bernoulli equation

$$\frac{\Delta p}{\rho} + g\Delta z + \tfrac{1}{2}\Delta v^2 = 0$$

for *steady* flow of *incompressible* and *inviscid* fluid

If the fluid is viscous

The conservative energies are not constant along a streamline, there is a dissipative energy (energy loss).

$$\frac{\Delta p}{\rho} + g\Delta z + \tfrac{1}{2}\Delta v^2 = \text{ energy } \textit{loss} \text{ per unit mass of fluid}$$

for *steady* flow of *incompressible* and *viscous* fluid

Restrictions on Bernoulli equation

The conservative energies are not constant along a streamline, if there are devices that can deliver or consume the energy, such as pump or turbine.

$$\frac{\Delta p}{\rho} + g\Delta z + \tfrac{1}{2}\Delta v^2 = \ - \text{shaft energy } \textit{by device} \text{ per unit mass of fluid}$$

Bernoulli equation may not be applied *across* the streamline, except for *irrotational* flow (where no rotational effects).